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When is a Physical Concept born? The Emergence of 'Work' as a Magnitude of Mechanics

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Abstract The physical magnitude 'work' has a long history. It emerged when two different practices, performed during the whole eighteenth century, met each other. The first was theoretical, practiced by philosophers and mathematicians, and was related mainly to the 'living forces' (vires vivae). The second was empirical, practiced by engineers, and was related to the work and the effectiveness of the motor engines. In both activities, the products 'weight (or force) multiplied by height (or displacement)' were used for calculations. Can we regard that these products constitute a well defined physical magnitude and are anticipations of the magnitude 'work'? Modern historians of science assert that 'work', as a magnitude of mechanics, was created in France, at the beginning of the nineteenth century. Why? In order to examine these issues, a brief survey into the history of the relevant ideas will be done, and a number of characteristics, acquired by the magnitude of 'work' through the historical process of its construction, will be presented. These characteristics may help to depict a magnitude, in general, which is autonomous and embedded in a physical theory. Finally, from the historical data concerning the history of 'work', some educational implications will be considered.

1 Introduction

The emergence of the physical magnitude 'work' occurred when two different practices, performed during the whole eighteenth century, met each other. The first was practiced by philosophers and mathematicians, and had to do with theoretical calculations related to the 'principle of virtual velocities' and the dispute concerning the 'living forces' (vires vivae). The second was performed by engineers, and had to do with practical calculations related to the work and the effectiveness of the motor engines (principally water-wheels and steam-engines). In both practices, philosophers, mathematicians and engineers were using for calculations the products 'weight multiplied by height' or 'force multiplied by

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displacement'. The physical measure 'work' emerged in the nineteenth century from these practices and these products.

This paper has to do with three issues. First, whether these products ('weight multiplied by height' or 'force multiplied by displacement') constitute an autonomous physical magnitude or can be regarded as first forms of the physical magnitude 'work', and whether we can use the term 'work' to signify them. Second, during the eighteenth century, mathematicians like d' Alembert, Euler, Lagrange etc., proposed various theoretical reformulations of Newtonian mechanics, expressing it in an algebraic form.[1] In their work, we can find mathematical relations describing the conservation of the 'living forces' (where the products of force multiplied by displacement were playing an important role), and these mathematical relations exhibit a great syntactical (mathematical) similarity with contemporary energy relations. Can we conceive these mathematical relations as energy relations or anticipations of energy relations? In order to investigate these issues, a brief survey into the history of the relevant ideas will be done. Also, picking up some clues from the philosophical discussion on the meaning of scientific terms, a number of characteristics, that the physical magnitude 'work' acquires during the historical process of its creation, will be outlined. These characteristics may portray an autonomous physical magnitude in general which is embedded in a theory. Third, looking at the details of the historical analysis, some possible educational implications for the teaching of the concept 'work' will be considered. These of course have to be tested in practice (in the classroom) and need further research.

2 Theoretical Calculations

Products 'weight (or force) multiplied by displacement (or speed)' were for long used in a principle of statics which in the eighteenth century was called 'principle of virtual velocities'. According to the eighteenth century formulation of the principle, if we give a small (virtual) movement to a system of bodies in equilibrium, the product of the moving force by its virtual velocity (or displacement) will be equal to the product of the resistance by the corresponding virtual velocity (or displacement).[2] The principle firstly appeared in the Aristotelian *'Mechanical Problems'* (Aristotle 1980, 3, 850b), showed up in medieval treatises on statics (Duhem 1991, pp. 75–113), was used by Galileo and Descartes (Galileo 1960, pp. 148–155; Descartes 1996, tom. I, pp. 435–448).

In 1637, for example, Descartes wrote the small treatise *'An explication of machines with which one can raise a very big weight with a small force' (Explication des engines par l' ayde desquels on peut, avec un petit force, lever un fardeau fort pesant)*, where he produced all theory of statics from the single principle that the 'force' needed to raise a weight depends both on the weight and the height of elevation.

> The invention of all these machines is based on a single principle: the force which can raise a hundred-libres weight, for example, to the height of two feet, can also raise a two-hundred-libres weight to the height of one foot, or a four-hundred-libres weight to the height of half a foot etc., provided that the force remains the same. And this principle must be accepted, if one believes that the effect should always be

[1] Newton had used Euclidean geometry.

[2] Since time was the same, the ratio of the velocities was equal to the ratio of the displacements, and the two options of the principle were equivalent.

> proportional to the action needed to produce it. So, if it is necessary to use an action in order to lift a weight of a hundred libres into a height of two feet, the same action would lift a weight of two hundred libres to a height of only one foot. For it is the same to lift a hundred libres to the height of one foot and afterwards to lift a hundred libres again to the height of one foot than to lift a hundred libres to the height of two foots (Descartes 1996, tom. I, p. 437).

In this description of the principle of virtual velocities, products 'force multiplied by displacement' are only implicit and invisible. The principle took its final form by Varignon and Johann Bernoulli, in Varignon's book *'New Mechanics or Statics' (Nouvelle Mécanique ou Statique)*, published at 1725 (Maggie 1969, pp. 48–50). In the nineteenth century, Coriolis reformulated it to the 'principle of virtual work' (Coriolis 1829, pp. 11–12).

In the context of the dispute concerning the 'force' of a moving body,[3] known as the 'vis viva controversy', natural philosophers and mathematicians were also using the product "weight (or force) multiplied by height (or displacement)" in order to calculate the measure of this 'force'. The term 'force of a moving body' was introduced by Descartes and his followers, and had to do with the result of the body's collisions. 'Force' was proportional to the body's 'quantity of motion', i.e. mass[4] times velocity ($m{\cdot}v$).[5] The overall 'force' (quantity of motion) in the Universe was constant[6] (Descartes 1996, vol. IX, *Principia Philosophia*, §36–40). For Leibniz and his followers,[7] on the other hand, a moving body's 'force' ('living force') was proportional to the body's mass and the square of its velocity (i.e. proportional to ($m{\cdot}v^2$). The total 'living force' in the Universe was also constant.[8] 'Living forces' were produced by the actions of the 'dead forces', i.e. the forces of statics (Leibniz 1971, 1989a, b). Leibniz and his followers estimated the cause or the result of this 'force' by the products 'weight multiplied by height' or 'pressure [force] multiplied by displacement' (Leibniz 1989a, pp. 300–301; Leibniz 1989b, pp. 442–444; Lindsay 1975, pp. 123–128). These products, however, both in the 'principle of virtual velocities' and in the case of the 'living forces' were not independent entities, and were only employed as auxiliary tools for calculations.

Leibniz, for example, in his article '*Specimen Dynamicum*', published in *Acta Eruditorum* in 1695, in order to show the incorrectness of the Cartesian 'force', calculated the 'forces' of two bodies that were moving downwards, using the magnitudes (weights-masses) of the bodies and their vertical displacement.

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[3] In order to distinguish this 'force' from Newtonian force, it will be written within inverted comas.

[4] Descartes was referring to the body's magnitude, a non-differentiated quantity of volume-weight-quantity of matter. The term 'mass' showed up after Newton.

[5] Without expressing them explicitly as products. The multiplication of two quantities that were not pure numbers was in the seventeenth century mathematically forbidden (Ravetz 1961).

[6] In continental Europe, this quantity was not considered to have direction, i.e. (in modern terminology) it was not a vector.

[7] Johann and Daniel Bernoulli, Christian Wolf, Samuel König, madame du Châtelet etc.

[8] In the case of a body moving upward, the ostensibly disappearing 'living force' was, in some way, piling up in the produced result, and could be regained during the body's fall (Leibniz 1989a, pp. 299–301). There were some cases, however, such as non-elastic collisions, where 'living force' appeared to vanish for good. Leibniz and his followers put forward the speculative and unconvincing argument that the seemingly disappearing 'living force' was actually transferred to the tiny particles which constituted all everyday bodies, asserting that although 'force' was lost for the colliding bodies, it was not in fact lost for the universe (Leibniz 1916, pp. 669–670).

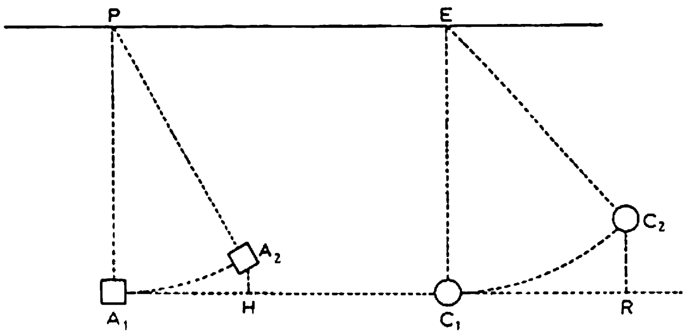

Fig. 1 Leibniz: Quantity of motion as the measure of ‘force’ leads to perpetuum mobile

> I assume it to be certain, however, that nature never substitutes for forces something unequal to them but that the whole effect is always equal to the full cause… If it were true, as men are commonly persuaded, that a heavy body A of magnitude 2 (for this is now our assumption) and velocity 1, and a heavy body[9] C with magnitude 1 and velocity 2, are equipollent to each other we should be able to substitute one for the other with impunity—a thing which is not true. For let us assume that A, with magnitude 2, has acquired a velocity of 1 in its descent A_2A_1 from the height A_2H, which is 1 foot (see Fig. 1); and then let us substitute for it, as it exists on the level of A_1, the weight C (which they claim is equipollent to it) with magnitude 1 and velocity 2, which ascends to C_2, a height of 4 feet. Thus, merely by the descent of a 2-pound [librae] weight from the height of 1 foot A_2H, we have, by substituting its supposed equipollential, brought about a rise of 1 lb [libra] to 4 feet, which is double the former effect. Therefore we have gained this much force or achieved a perpetual mechanical motion, which is absurd (Leibniz 1989b, pp. 443–444).

Some years later, Johann Bernoulli, Leibniz’s most distinguished follower, in his ‘*Hydraulica*’, published in 1732,[10] calculated the ‘living force’, that a body acquires when pushed by a stretched elastic, through the integral $\int pdx$, where p is the variable motive force of the elastic and dx the infinitesimal displacement.

> A variable motive force is one of which the intensity is changed while acting. Thus, for instance, the force of a stretched elastic has a greater intensity, and as a consequence impresses on the body to be propelled a greater accelerative force at the beginning than during the progression of relaxation. From the above, these Rules result:
> Let the space traveled by a body $= x$
> the mass of the propelled body $= m$
> the motive force within the limit of the region traveled $= p$
> the velocity acquired $= v$
> the time trough $x = t$

[9] The language of physical philosophy had not yet got rid of the Aristotelian distinction between (absolute) heavy bodies, which fall downwards, and (absolute) light bodies, which ascend upwards.

[10] The book was actually published in 1743, but Johann Bernoulli had put 1732 as the year of the publication, in order to gain priority over his son’s (Daniel) ‘Hydrodynamica’ (see the preface of H. Rouse in Bernoulli 1968).

> Hence $dt = dx/v$; there will be pdt/m or $pdx/mv = dv$, and therefore $\int pdx = 1/2mvv$, which is well known. (Bernoulli 1968, p. 354)

It is obvious, both in the writings of Leibniz and in the writings of Johann Bernoulli, that products ‘weight or force multiplied by height or displacement’ are simply a calculating tool. We can see the same thing in almost all the literature on the ‘living forces’ (see for example du Châtelet 1988, pp. 444–446).

Cartesians and Newtonians attacked the leibnizian views. Cartesians were supporting the ‘quantity of motion’ as a conserved quantity in the Universe, whereas Newtonians were rejecting completely that force (meaning Newtonian force) was conserved. Though the conflict faded over the years without resolution, in reality Leibnizians lost the battle, and ‘living forces’ were put aside, at the margin of physical philosophy, until the end of the eighteenth century. In fact, ‘living forces’ were only used in some special cases, such as the elastic collisions (Iltis 1970, p. 140).

The eighteenth century mathematicians who created the great theoretical reformulations of Newtonian mechanics (d’ Alembert, Euler, Lagrange etc.) employed mathematical expressions and relations syntactically (mathematically) similar with modern mathematical representations of the energy concepts and relations (d’ Alembert 1758, pp. 252–258; Euler 1753; Lagrange 1788, pp. 206–227). This similarity led some earlier historians of science, such as Jurdain (1913), Lindsay (1975) etc., to regard these expressions and relations as anticipations of energy concepts and relations and to signify them accordingly. Syntactical similarity however is misleading. These mathematicians were giving different ontological content and meaning to these expressions and relations, as well as different significance.

D’ Alembert, for example, in the case of two bodies with masses A and a, connected by a rod Aa, and under the influence of (external) ‘moving forces’ (puissances motrices), proved that

$$AVV + avv = ABB + abb + \int 2A \cdot AD \cdot CA + \int 2a \cdot ad \cdot ca$$

where B, b were the bodies’ initial velocities, and V, v their final velocities (VV means V^2), AD, ad represented the magnitudes of the ‘moving forces’ that acted on the bodies, and CA, ca represented the bodies’ displacements in the direction of the ‘moving forces’ (see Fig. 2).

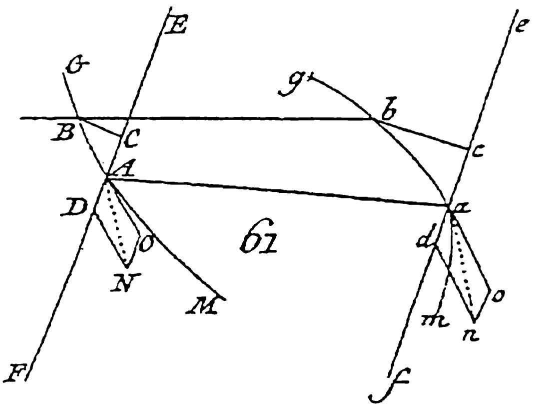

Fig. 2 D’ Alembert on the ‘living forces’

D' Alembert interpreted the integrals $\int 2A \cdot AD \cdot CA$ and $\int 2a \cdot ad \cdot ca$ as the additional 'living forces' which the (external) 'moving forces' imposed on the bodies A and a. If the 'accelerating force', activating the bodies, was gravity g, these integrals would become $2Agx$ and $2agz$, where x and z were the vertical components of the curves the bodies described. D' Alembert interpreted $2Agx + 2agz$ as the 'living force' which the sum of the two weights produced, descending a height equal to the center of gravity's descent (d' Alembert, pp. 256–269).

Lagrange, introduced two mathematical functions, in order to derive the equations of motion for a system of bodies, under the action of any forces. The first was function T, which was the one half of the system's 'living force', that is

$T = \int ((dx^2 + dy^2 + dz^2)/2dt^2)m$, where x, y, z are the Cartesian coordinates of the body m. The integration runs for all bodies m. The second was function $V = \int \Pi m$, that is the sum of all functions Πm for all the system's bodies, where $\Pi = \int Pdp + \int Qdq + \int Rdr + \ldots$, and P, Q, R,… were the 'accelerating forces' (accelerations with characteristics of force, that is forces per unit mass or intensities of the forces) that act on the body m, and dp, dq, dr,… were the infinitesimal displacements of the body m along the directions of the distances p, q, r,…, from the centers of the 'accelerating forces' P, Q, R,… Function V, according to Laplace, represented the influence of the 'accelerating forces' on the system's total 'living force'. Using these functions, Lagrange could express elegantly the equations of motion for the system. Also he could derive the equation $\int ((dx^2 + dy^2 + dz^2)/2dt^2 + \Pi)m = F$ where F is a constant. This equation, according to Lagrange, describes the principle known as the 'conservation of the living forces' (Lagrange 1788, pp. 206–227).

D' Alembert's integrals $\int 2A \cdot AD \cdot CA$ and $\int 2a \cdot ad \cdot ca$, however, were only tools for the calculation of the additional 'living force'. Lagrange's expressions T and V were simply mathematical functions without ontological or directly empirical content. Neither had they any direct connection with experience nor were they regarded to represent some physical entity (as was the case with the energy expressions in the nineteenth century). 'Living force', for example, was not in fact considered to be conserved, since it seemed to disappear at non-elastic collisions. When Leibniz introduced it, the concept had a strong metaphysical-ontological dimension. For Leibniz 'force' was one of the main entities implanted in the universe by the Creator. A kind of this 'force' was the 'living force' (Leibniz 1989b). Soon, however, with the exception of some German philosophers such as Christian Wolf, this dimension was put aside. In the work of d' Alembert and Lagrange, no sign of this metaphysical dimension was apparent (d' Alembert 1758; Lagrange 1788). Equation $\int ((dx^2 + dy^2 + dz^2)/2dt^2 + \Pi)m = F$ also, was not but a mechanical theorem with very limited applications. Neither had it the metaphysical connotations nor the universality the energy relations would have in the nineteenth century.

These remarks are summarized in Table 1.

3 Empirical Calculations

The second group of activities concerning the product 'force multiplied by displacement' were performed in the area of engineering and had to do with the motor engines (mainly water-wheels and steam-engines), and the attempts of the engineers to study them. A systematic analysis of water-wheels, which was done by the French mathematician-engineer Antoine Parent at the beginning of the eighteenth century, focused on the impact

Table 1 Anachronistic and contextual interpretations

Century	Historical agents	Mathematical expression or relation	Anachronistic interpretation	Contextual interpretation
Seventeenth century	Galileo, Descartes, Varignon etc. (statics)	Force multiplied by displacement	Anticipation of 'work'	A tool for calculations
Seventeenth and eighteenth century	Leibniz and his followers (dynamics)	Force multiplied by displacement	Anticipation of 'work'	A tool for calculations
Eighteenth century	D' Alembert (dynamics)	$\int 2A \cdot AD \cdot CA$ or $\int 2a \cdot ad \cdot ca$	Anticipations of 'work'	Tools for calculations
Eighteenth century	Lagrange (dynamics)	$V = \int \Pi m$ where $\Pi = \int Pdp + \int Qdq + \int Rdr + \ldots$ $\int ((dx^2 + dy^2 + dz^2)/2dt^2 + \Pi)m = F$	Anticipations of the energy concepts and the energy relations	Mathematical functions and relations without ontological or directly empirical content

forces the water exerted against the wheel's blades, and ignored or underestimated the action of the water's weight (Parent 1704, pp. 325–333).

Parent, for example, examining theoretically an undershot waterwheel which raised weights, assumed that the fluid (water) had velocity V and that the maximum weight that the wheel could keep immovable was P. If the wheel was used to raise a smaller weight p, this weight would ascend with velocity u. Parent claimed that the useful (produced) result would be pu, whereas the whole result the stream could give would be PV (Parent 1704, pp. 325–328).

> The result produced by the fluid…, in a given time, does not only increases as much as the weight increases, but also in proportion to how quick the weight ascends. I call *general result* of the fluid the product of the solid weight [p] which is raised multiplied by the velocity [u] of this weight. But I call the product of the weight P multiplied by the velocity of the fluid [V] *natural result* (Parent 1704, p. 326).

The majority of water-wheel studies in the eighteenth century, following Parent's mode of analysis, examined water-wheels statically, focusing on the momentary actions of forces, and did not calculate the wheel's accumulated result in a given time interval (Desaguliers 1744, pp. 449–453; Triewald 1734; Hutton 1796 entry MILLS). Desaguliers, in the second volume of his book '*A course of experimental philosophy*', published in 1744, described in detail the analysis the engineer Henry Beighton had done on an overshot water wheel in Nuneaton, Warwickshire. The wheel had 30 buckets and his diameter was 16 feet. The water fell on the buckets from the height of 7.5 feet with a velocity of 1,350 feet per minute and the wheel made 8 revolutions per minute. Beighton calculated the flow of the water to be 1206 ale gallons per minute, which gives 5.02 ale gallons or nearly 50 lb wt (pounds of weight) in each bucket. Adding up these 'forces' and taking into consideration the effect of their horizontal distance from the wheel's axle (finding what he calls 'statical weights'), he calculated that their total sum (from all full buckets), every moment, was 402.1 lb. This was the 'forces' due to the water's weight. After that Beighton calculated the 'forces' due to the impact of water on the puddles (buckets). Since one cubic inch of water weights 58 oz, and there were always falling 3,780 cubic inches of water, the 'force' impressed on the wheel, were it on the tangent of the wheel, would amount to 137 lb. But water was striking obliquely on the puddles, nearly to an angle of 45°. Considering this and checking his calculations with experiments (as he says), he found a total 'force' of 462 lb (Desaguliers 1744, pp. 449–453).

It is obvious that the analysis was focused on momentary forces, that is impact forces, and static forces. Although the results of his two calculations were comparable, Beighton insisted on the significance of the water's impact on the puddles and on the height of waterfall.

> This water-mill is by most people accounted as good a one, as any country affords, for dispatching as much business in the time, and doing it well… However this goes well, it is a very agreeable height 16 feet, the fall considerable 7½ feet, for the wheel being made 20 feet high in the place it stands, it would not have been capable of doing so much business: Of so much more service is the impulse, stroke, or momentum of the water, than is its bare statical weight (Desaguliers 1744, p. 453).

The British engineer John Smeaton and the French engineer Jean Charles Borda calculated the accumulated result of the water-wheels, or the 'power' capable to produce this result, using the product 'weight multiplied by height' (Smeaton 1759; Borda 1770). Smeaton, in his '*An experimental enquiry concerning the natural powers of water and*

wind to turn mills, and other machines, depending on a circular motion', published in 1759 in the *Philosophical Transactions*, studied water-wheels and measured their effect by the product 'weight multiplied by height' (Smeaton 1759).

> The word *power*, as used in practical mechanicks, I apprehend to signify the exertion of strength, gravitation, impulse, or pressure, so as to produce motion: and by means of strength, gravitation, impulse, or pressure, compounded with motion, to be capable of producing an effect: and that no effect is properly mechanical, but what requires such a kind of power to produce it.
> The raising of a weight, relative to the height to which it can be raised in a given time, is the most proper measure of power; or, in other words, if the weight raised is multiplied by the height to which it can be raised in a given time, the product is the measure of the power raising it;…(Smeaton 1759, p. 105).

Smeaton's and Borda's work, however, was not immediately acknowledged, probably because they used the disrespectful 'living forces' (Reynolds 1973, pp. 487–505, 515–520).

Another kind of motor engine, spreading all around Britain during the eighteenth century, was the steam-engine. Originally, steam-engines were build to move water-pumps for the drainage of mines, and engineers were seeing them principally as strong pumps. Their 'duty' (effectiveness) was measured by the amount of water they could raise to a certain height in a certain time interval or with the consumption of a certain quantity of coal. More specifically, steam-engines were evaluated by the number of pounds (lbs) of water they could raise one foot high in a minute or with the consumption of one bushel[11] of coal.

In 1769, for example, Smeaton, with the help of William Brown of Throckley, examined about 100 Newcomen steam engines, mainly in the North of England, and estimated that in the average they raised 5.59 millions of pounds of water 1 ft high by the consumption of 1 bushel of coal. In 1772, Smeaton redesigned and improved a Newcomen engine at Long Benton Colliery, Northumberland, and the engine attained a 'duty' of 9,450,000 lb raised 1 ft high by the consumption of 1 bushel of coal (Dickinson 1963, pp. 61–62).

This estimation ('duty') was directly connected with another measure of the steam-engine's 'duty': the 'horse' or 'horsepower'.[12] A 'horse', according to the value fixed by Watt (at least in Britain), was equivalent to 33,000 lbs raised 1 ft high in 1 min (Cardwell 1967, pp. 214–216; Cardwell 1971, p. 33; Hills and Pacey 1972, pp. 28–31; Hills 1989, pp. 88–94).

The engine's 'duty' was originally connected to the engine's product (i.e. the raising of water), and was not a universal measure for estimating the performance of steam-engines or motor engines in general. This quantity remained a measure of steam-engines' capacities, even when steam-engines were used to move other engines (e.g. the machinery in textile mills) or vehicles. For many years, however, this measure would remain an empirical tool, restricted within the area of practical engineering, without connections with other quantities, such as 'living force', or with a more general theory (Kanderakis 2007, p. 53).

[11] A unit of capacity (volume) for dry cargo. 1 bushel = 8 gallons (Hutton 1796, entry MEASURE).

[12] Since steam-engines usually replaced horse-driven pumps, their 'duty' was measured by the number of horses they might replace (Hills and Pacey 1972, p. 30).

4 The General Theory of the Moving Engines and the Creation of the Physical Magnitude "Work"

The emergence of 'work' as a physical magnitude embedded in the theory of mechanics came about when the two previous practices were met in a general theory of the moving engines, whose main concept was 'work'. This theory emerged in post-revolutionary France at the beginning of the nineteenth century, within the world of French higher technical education, and under the effort of the French society to overcome its technical backwardness compared to Britain. Although a large number of engineers were engaged, it was Navier, Coriolis and Poncelet, engineers with high academic credentials, and teachers in French 'Grands Écoles', who played the most significant role.[13]

In 1819, Louis Navier edited a new publication of Bellidor's '*Architecture Hydraulique*' (an old textbook for engineers), presenting all new data in numerous and extended footnotes and additions. In this new publication, he sketched a general theory of motor engines, with 'living force' and 'quantity of action' as central magnitudes. The 'quantity of action' measured the work or the result of the engine, and was equal to the product of the force (or 'pressure' or 'effort') exerted by the engine multiplied by the distance covered by its point of application in its direction. Until then, Navier pointed out, the work of different mills was being calculated by the different outputs they had produced, and a comparison between them was impossible. 'Quantity of action' could be used as a common measure of their outputs, a kind of 'mechanical currency', suitable for any kind of engine and any kind of work (Bellidor 1819, pp. 103–122, 376–395).

> Let us suppose a person who has a wheat mill, and wants, through some changes in the mechanism, to turn it to saw mill. He can not estimate the advantage or the disadvantage of this alteration, the quantity of wood that the mill will give from the quantity of flour that it produces now. This calculation is absolutely impossible, at least if we will not find a common measure for these amounts of work, which they are so different in nature. This example is adequate to show us the need for the establishment of a kind of *mechanical currency*, if we can call it so, by the help of which we will be able to estimate the quantities of work needed to produce all kinds of goods (Bellidor 1819, p. 376).

The creation of the 'quantity of action' was based on the raising of a weight, which was the model for every other work. According to Navier, any work, whatever its nature might be, is always equivalent to the raising of a weight, not only in thought but in reality (using ropes and pulleys), (Bellidor 1819, pp. 377–378). Using these concepts, Navier calculated the produced result in a series of cases concerning work, such as the average daily work of a man, and analyzed a number of motor engines. For example, using the 'living force' and the 'quantity of action', he analyzed the performance of an overshot water-wheel, and calculated the maximum 'quantity of action' that the wheel could produce (Bellidor 1819, pp. 383–384).

In 1829, Gaspard Coriolis, in his textbook '*Du Calcul de l' Effet des Machines*' *(Calculating the Result of Machines)*, introduced two novelties: the name 'work' (travail)

[13] Among them we can include Lazare Carnot. Carnot created a general theory of machines whose central magnitude was the 'moment of activity' (moment d' activité), that is the product of the force acting on a body by the body's velocity and time (Carnot 1783, pp. 68–74). Carnot's analysis, however, was too theoretical and obscure (at least for the engineers) and had little impact on later developments (Gillispie 1971, p. 101; Séris 1987, pp. 372–374).

for the main magnitude of the moving engine's theory, and the modification of the magnitude of 'living force' from mv^2 to ½ mv^2, in order to be exactly equivalent to 'work' and not simply proportional to it (Coriolis 1829, pp. 14–20). Coriolis picked up the name 'work' in order to connect this measure with the use of the term work in everyday language. 'Work' was defined as the integral $\int Pds$ of the force P acting in the direction of the infinitesimal displacement ds along the displacement.

> We propose for the quantity $\int Pds$ the name *dynamical work* or simply *work* (*travail*)… We will not confuse this name with any other mechanical name, and it seems suitable to give an exact description for this thing, keeping, in the same time, its common acceptance as the conception of physical work (Coriolis 1829, p. 17).

This magnitude was used to measure the work (labor) of motor engines, men, and animals. In a system of bodies, the equation of the 'living forces' was:

$$\sum \int Pds - \sum \int P'ds' = \sum pv^2/2g - \sum pv_o^2/2g$$

where P is the moving force, P' the resisting force, g the acceleration of gravity, v_o and v the initial and final velocities, p the body's weight, and p/g the body's mass. According to the new terminology this equation can be described as follows:

> In all systems of moving bodies, the difference between the sum of the quantities of work done by the moving forces and the sum of the quantities of work done by the resisting forces, in a given time, is equal to the change of the sum of the living forces of all the system's masses, in the same time (Coriolis 1829, p. 19).

The unit of 'work' was the 'dynamode': the 'work' of a force of 1,000 kilograms when its point of application was displaced 1 m (Coriolis 1829, p. 33). Coriolis calculated the 'work' which was done in a vertical and a horizontal transportation of a weight, the 'work' which was produced by an expanding gas or steam, the 'work' that a moving fluid transmits to another body, etc. (Coriolis 1829, pp. 36–90).

A similar general theory of the moving engines (or an 'industrial mechanics'), more popularized and adapted to the level of practical engineers, was presented in 1826 by Jean Victor Poncelet into the lithographed book '*Cours de mécanique appliqué aux machines*', and in 1829 into '*Cours de mécanique industrielle fait des artistes et ouvriers messins, pendant les hivers de 1827 à 1828, et 1828 à 1829*' which was reprinted in 1839. This second textbook was printed again in 1870 under the title *'Introduction a la Mécanique Industrielle Physique ou Expérimentale'* (*Introduction to industrial mechanics, physical or experimental*), with the addition of all new data in the footnotes (Chatzis 1998). Poncelet defined 'work'[14] as the product of the acting force multiplied by the displacement in the force's direction, and noted that 'work' presupposes both a resistance to be overcome and a space to be travelled. According to Poncelet, a (steady) movement without resistance, due to the inertia of matter, does not need an effort, and does not require work. On the other hand, if we apply an effort (force) or keep a weight without movement, we do not really work, because we can always replace the acting agent with a stationary support (rope, pillar etc.), (Poncelet 1870, pp. 78–80). When the only resistance was the moving body's inertia, 'work' was transformed into 'living force'.

[14] Initially Poncelet was using Navier's 'quantity of action', but with the encouragement of Coriolis he accepted the name 'work' (Darrigol 2001, p. 317; Poncelet 1870, p. 2).

> The expression living force, which is used to signify the product *(P/g)* $\cdot\ v^2$,…, properly speaking, is not a force at all. No more than the quantity $P{\cdot}H$, named quantity of action or quantity of work. It is simply the result of the action of a moving force or pressure, expressed as a weight, which is used, for a longer or a shorter time, to overcome the inertia of a body's matter and to impress on this body a given movement, a given velocity. According to this view, living force is in fact the *dynamical result* of the moving force, or better, since $(P/g) \cdot v^2 = 2P \cdot H$, the double of this result (Poncelet 1870, pp. 117–118).

In a vertical transportation of a weight, according to Poncelet, we could calculate 'work' through the product of the weight multiplied by the vertical displacement. This is not the case in the horizontal transportation, however. In this case, we should not multiply the displacement by the weight which is transported, as some former engineers were doing (e.g. Coulomb 1799), but by the resistance (friction) we confront. As a unit of 'work' Poncelet took 'one kilogram raised one meter high' or 1 kgm. Poncelet used 'work' to analyze all sorts of machines as well as various physical phenomena. For example, he calculated the 'work' that solar heat spends annually per square league of the Earth's surface in order to create rain, the 'work' of the expansion of gases and the 'work' produced by the expansion of the steam inside a steam-engine (Poncelet 1870, pp. 64–85, 96–120, 217–222).

In the end, lectures and lessons given by Navier, Coriolis and Poncelet, their textbooks, and lectures and books written by their students and followers (see for example Bélanger 1864) spread the new ideas within the world of engineers, and finally within the world of rational mechanics (Darrigol 2001, pp. 311–317). At the same time, French science, as Arago and Dupin noted, made a shift of focus from the ideal laws of an abstract world to the needs of the arts and the industry (Arago and Dupin 1827).

> A few years ago, France was accused that it was giving to the educated world only theories of mechanics undoubted sublime but presented as ideal laws of an abstract world, which had almost nothing in common with the reality of work in the workshops and the industries. Our teachers in the École Polytechnique have taught us the way to bring science closer to the needs of the arts, and to draw from its important discoveries consequences easily and frequently applicable to all the operations of the industry (Arago and Dupin 1827[15], p. 531).

In Britain, a similar transition in theoretical mechanics took place almost 10 years later, through two different routes. The one occurred through the evolution and systematization of the empirical practices the engineers were using in order to measure and compare their engines. The other came through getting and assimilating the French studies about these matters (Smith and Wise 1989, pp. 58–65; Smith 1998, pp. 31–39; Wise 1989, pp. 226–229 and 1990, p. 244).

5 When Does a Mathematical Expression Become Physical Magnitude?

Until the beginning of the nineteenth century, the products 'weight multiplied by height' or 'force multiplied by displacement' were used as calculating tools and we cannot regard

[15] Poncelet had already submitted his '*Cours de mécanique appliqué aux machines*' to the Académie des Sciences.

them as expressions of an autonomous physical magnitude. Modern historians of science assert that 'work' emerged as a physical magnitude only in the beginning of the nineteenth century with the work of Navier, Coriolis and Poncelet (Chatzis 1997; Darrigol 2001). Why? What are the characteristics that a well defined independent physical magnitude, embedded in a theory or a branch of classical physics, has? Surveying the history of the creation of 'work' as a concept of mechanics, and picking some clues from the philosophical discussion on the meaning of theoretical terms, we can single out a number of characteristics that an independent physical magnitude, like 'work', gets through the historical process of its construction. These characteristics emerged from the details of the historical analysis of a certain case ('work'), and were not deduced from philosophical considerations based on a priori truths. Accordingly, more examples are needed to establish a more general conclusion. The characteristics are the following:

1. The physical magnitude is well defined, and the conditions under which we can use it are determined. Coriolis, for example, in defining 'work', explained that we had to regard the component of the force in the direction of the displacement than the total force. As we have already seen, Poncelet criticized Coulomb, who calculated the work done by a man transporting a body horizontally by the product of the transported weight multiplied by its displacement (Coulomb 1799, pp. 398–408). According to Poncelet, we have to use the horizontal force which destroys resistance in order to calculate this work and not the body's weight (Poncelet 1870, pp. 83–85).
2. The physical magnitude has a name. As Roche remarks 'the creation of a distinct and accepted technical term was an important step in establishing an independent general concept'[16] (Roche 1998, p. 98).
3. It has units with a name (for example '1 kgm'). Units and their names increase the magnitude's significance.
4. It has generality and universality, that is, we can use it in every place and time and not in special circumstances only. An example of local applicability (non-universality) was the use of the output of a complex machine in order to measure its work. For example, the flour produced by a grain-mill was used to measure the work of its machinery; the quantity of planks produced by a saw-mill was used to measure its work etc. In the end, a certain output—the result of a steam-pump's work—became the model-output for measuring the work of all engines (and all actors).
5. At least in classical physics, we can use it for calculations in certain experimental situations or practical applications, consequently it has empirical content or empirical meaning (Achinstein 1964, pp. 501–502; Hempel 1966, pp. 85–86; Baltas 1990, pp. 295–296). According to Achinstein part of a term's meaning is 'knowing the range of application of the term, i.e. the sorts of situations in which it can be employed' (Achinstein 1964, p. 502). According to Baltas, the concepts of physics are the indisputable 'clothing' of physical phenomena, in order to be understood as physical and not as natural phenomena,[17] always remaining anchored to them. These concepts are interpreted by the phenomena they describe, and their connection to the

[16] Roche refers to the introduction of the term 'momentum' by the Renaissance mathematician Francesco Maurolyco. The term had been used from Antiquity in situations related to motion, weight, and the influence of a weight in general. Maurolyko used it to signify the turning action of a weight (Roche 1998, p. 98).

[17] Baltas refers here to classical physics. In modern physics there are phenomena accessible only through the conceptual system of physics itself, and the distinction between natural phenomena (apprehended through pre-scientific notions) and physical phenomena (apprehended through the concepts of physics) does not hold (Baltas 1990, p. 309).

phenomena determines the empirical component of their meaning (Baltas 1988, pp. 295–296).

6. It is connected with other magnitudes in a theory through mathematical relations that express principles or empirical laws. Consequently, it has systemic meaning or, to be more precise, its meaning has a systemic component (Hempel 1966, pp. 93–94; Hempel 1970, pp. 142–149; Baltas 1990, pp. 294–298). The notion of systemic meaning is found in many logical empiricists (and their successors) arguments. Achinstein, for example, maintains that part of the meaning of a scientific term 'may involve learning its role or roles in theory' (Achinstein 1964, p. 501). A kind of systemic meaning, necessary for the formulation of the incommensurability thesis, we also find in the post-positivistic, historical school of philosophy of science (Kuhn, Feyerabend etc.). As Feyerabend remarks, 'the meaning of a term is not an intrinsic property of it, but is dependent upon the way in which the term has been incorporated into a theory' (Feyerabend 1981, p. 74). Baltas is more explicit. According to him, physical concepts 'make physical sense (are employed in scientific explanations, in the design of experiments, in the interpretation of experimental results etc.) only as elements of a conceptual system', and their epistemic function (i.e. their meaning) always depends on the other concepts of the system, which are constitutively interrelated. The meaning of a concept, therefore, has also a holistic character, a systemic component, which is determined by the position occupied by this concept within the conceptual framework and its (mathematical) relations with the other concepts of the system[18] (Baltas 1990, pp. 296–297).
7. The historical agents (scientists), who created the new magnitude, were employing it as an important and autonomous quantity. This is (from the historian's point of view) the most significant characteristic.

It could be argued that the mathematical expressions $F{\cdot}s$ or $\int Fds$ have these characteristics only in the work of Navier, Coriolis, Poncelet and afterwards. In their lectures and books, the magnitude 'work':

1. is well defined, and the conditions for its use are well determined, both within the theory of the moving engines and within rational mechanics in general.
2. gets a name.
3. has well defined units with a name.
4. has general and universal applicability. Not only it applies to all kinds of motor engines, but it is also used in areas of mechanics beyond machines. In some cases it is even used for calculations beyond mechanics.
5. is the main magnitude for practical calculations as regards motor engines' effectiveness and the work done by engines, humans and animals. Consequently it is directly related to experience, i.e. it has empirical meaning.
6. is mathematically related with other magnitudes of mechanics. For example, it is related with 'living force', with the magnitudes of fluid mechanics when the 'work' of

[18] Arabatzis believes that this conception of meaning can be used as a historiographical and philosophical tool only if the conceptual system in question is fixed and coherent. If the system evolves, we cannot discriminate which of its changes transform the meaning of its terms. If the system is not coherent, it is not clear how the terms get their meaning. Arabatzis prefers a view about meaning which is based upon actual scientific practice. According to it, the meaning of a term amounts to the characteristics of the entities or processes to which the term refers, ascribed by the scientists who are using the term (Arabatzis 2006, p. 16). This approach does not contradict the way systemic meaning is used in this article, if as scientific practice we also take scientist's theoretical activity.

a gas's expansion is calculated (Poncelet 1870, pp. 217–222) etc. Consequently it has a systemic meaning.

7. Historical agents who created 'work', that is Navier, Coriolis and Poncelet, regarded it an important and independent quantity. In fact, it is the main magnitude in their theory for the moving engines.

Summarizing, it is suggested that only after the work of Navier, Coriolis and Poncelet, can we regard 'work' as an independent physical magnitude, embedded in the conceptual framework of mechanics.

6 Conclusion and Educational Implications

The product 'weight multiplied by height' or 'force multiplied by displacement' was used for over a hundred years in two different practices: in theoretical calculations related to the 'principle of virtual velocities' and the 'living forces' (a theoretical practice), and in practical calculations as regards the work and the effectiveness of motor engines (an empirical practice). In both activities, these products were used only as calculating tools, never got an autonomous existence, and cannot be regarded as expressions of an independent magnitude.

The emergence of 'work' as a magnitude of mechanics occurred when these practices converged in the context of the creation of a general theory for the moving engines. This convergence took place in France, at the beginning of the nineteenth century, within the world of the higher French technical education. Only then, can be argued, 'work' has a number of characteristics, acquired through the historical process of its construction, that they depict it as an autonomous physical magnitude, embedded in the theory of mechanics.

These considerations have also some educational implications as regards the teaching of 'work' in secondary education:

i. The long, difficult and complicated endeavor of engineers, philosophers and mathematicians to create the physical magnitude 'work' indicates the difficulty students might confront nowadays in order to assimilate this concept themselves. From science education literature we learn that: first, students estimating the work done by a man who lifts a body, tend to examine only the applied force and ignore the height of elevation (Driver and Warrington 1985) second, when a force is acting on a body, students appear to believe that the time of action determines the energy the body obtains, whatever the distance covered may be (McDermott 1984) and third, students have difficulty to interpret energy change problems in terms of 'work' and 'energy', and opt to describe them in terms of more direct characteristics of the system, such as forces (Driver and Warrington 1985). Although these misconceptions or alternative ideas do not recapitulate the history of the creation of 'work', this history can offer us many ideas to teach 'work'. In order to confront student's difficulties, we can employ, among other things, the arguments the pioneers used to persuade their peers. For example, in order to maintain that passive forces do not produce 'work', we can use Poncelet's argument that a force which keeps a weight without movement does not produces work, because we can replace it by a stationary support (a pillar or a rope), (Poncelet 1870, pp. 78–80).

ii. The physical magnitude 'work' was created to measure work (labor): firstly the work of motor engines, and secondly the work of humans and animals (horses). This primary function of 'work' usually does not show up in textbooks (nor probably in teaching), and 'work' is understood as a theoretical construction that has no relation to the everyday world

(see for example Marshall and Jacobs 2004, pp. 206–207). Science often creates new concepts in order to solve practical problems from everyday life or technology. Presenting these problems in the classroom, as well as the conditions under which the new concepts emerged (in a simplified-transformed version), could show students the necessity of the new concepts, and excite their interest.

iii. The emergence of 'work' as a physical magnitude can be understood as a three-stage procedure. Initially, engineers created a measure for a certain type of work: the elevation (by the steam-pump) of a certain weight to a certain height. Subsequently, this work was used as a model for measuring any kind of work done by motor engines. For example, it was used to measure the work done by steam-engines in order to move textile machinery. Finally, this measure was generalized to measure any kind of work done by any actor. For example, it was used to measure the work of men and horses. During this process, 'weight' became 'force' in general and 'height' became 'displacement in the direction of the force'. In addition to this, 'work' was connected (through mathematical relations) to the 'living forces' and some other magnitudes of mechanics. Accordingly, it got a systemic meaning, and was included in the conceptual framework of mechanics. This process gives us some clues for a new approach to teaching 'work', in a way more related to everyday life. The course might include the following steps:

a. introducing 'work' as a measure of a man's (or an elevating engine's) effort to raise a certain weight to a certain height,
b. extending this measure to the horizontal movement of a weight,
c. extending it to other kinds of processes (using an electrical toy car, for example, to transport horizontally a small box),
d. relating 'work' to other theoretical concepts (e.g. kinetic energy), and giving systemic meaning to it.

These ideas, however, in order to prove their applicability, have to be tested in practice (in the classroom). Consequently, further research is needed.

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